A CONDITIONAL APPROACH TO EXTREME EVENT ATTRIBUTION

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Slides, datasets etc.: http://rls.sites.oasis.unc.edu/ClimExt/intro.html

THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL

The Guardian, June 27, 2023

Current heatwave across US south made five times more likely by climate crisis

Latest 'heat dome' event over Texas and Louisiana, plus much of Mexico, driven by human-cause climate change, scientists find

A temperature display reading 99F (about 37.2C) in late afternoon in Houston, Texas, at the weekend. Photograph: Xinhua/Shutterstock

The record heatwave roiling parts of Texas, Louisiana and Mexico was made at least five times more likely due to human-caused climate change, scientists have found, marking the latest in a series of recent extreme "heat dome" events that have scorched various parts of the world.

Objectives

1. "Extreme event attribution" is an active field drawing much publicity (see in particular, the website of "World Weather Attribution")

2. My objective is to extend existing approaches, not contradict them

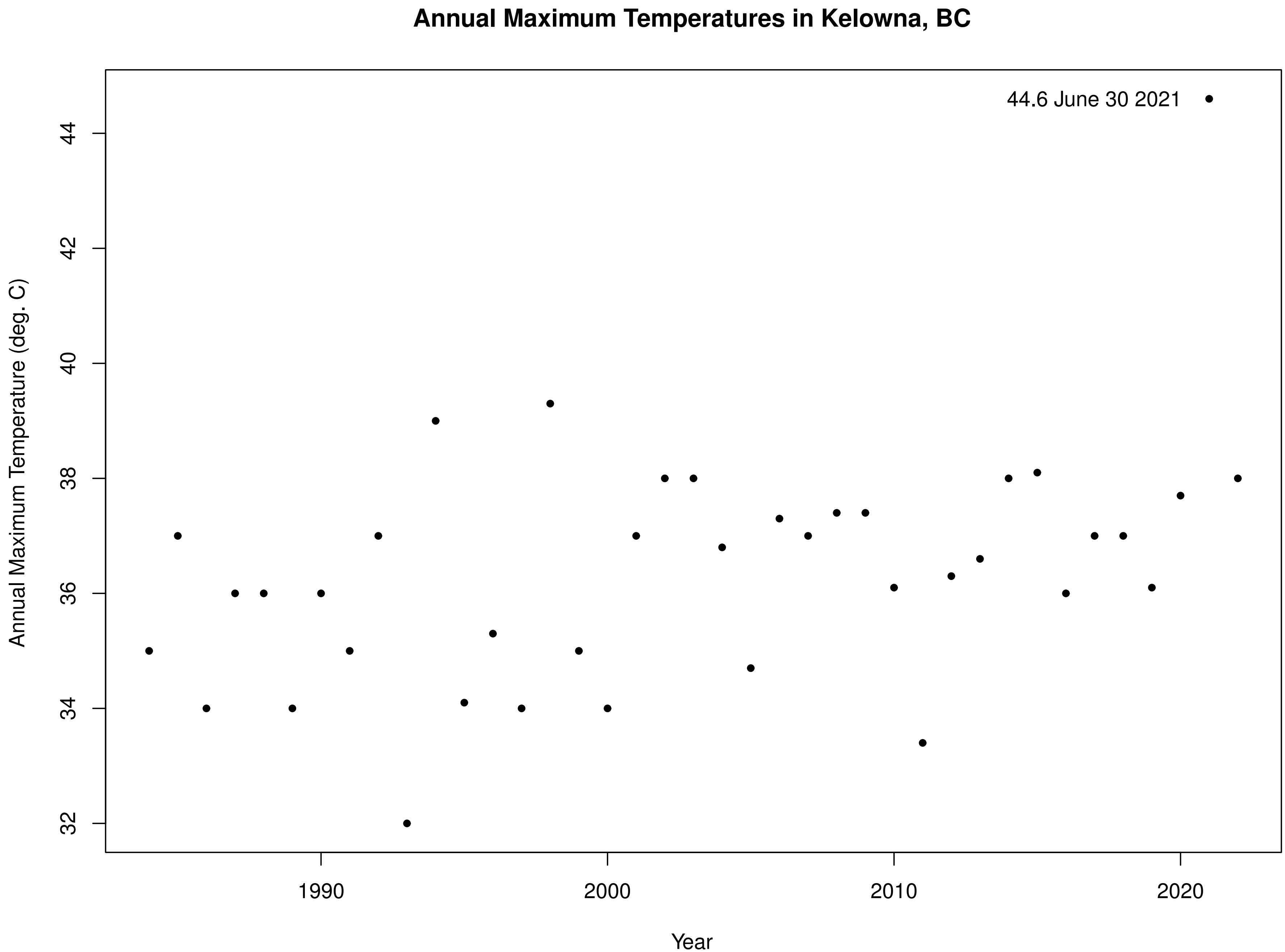
3. Acknowledging that dynamical methods will ultimately outperform statistical methods, but the latter are much quicker to calculate and provide an independent validation

4. Key idea of this talk: include a *conditioning variable* — some regional climate indicator at a more localized scale than global mean surface temperature

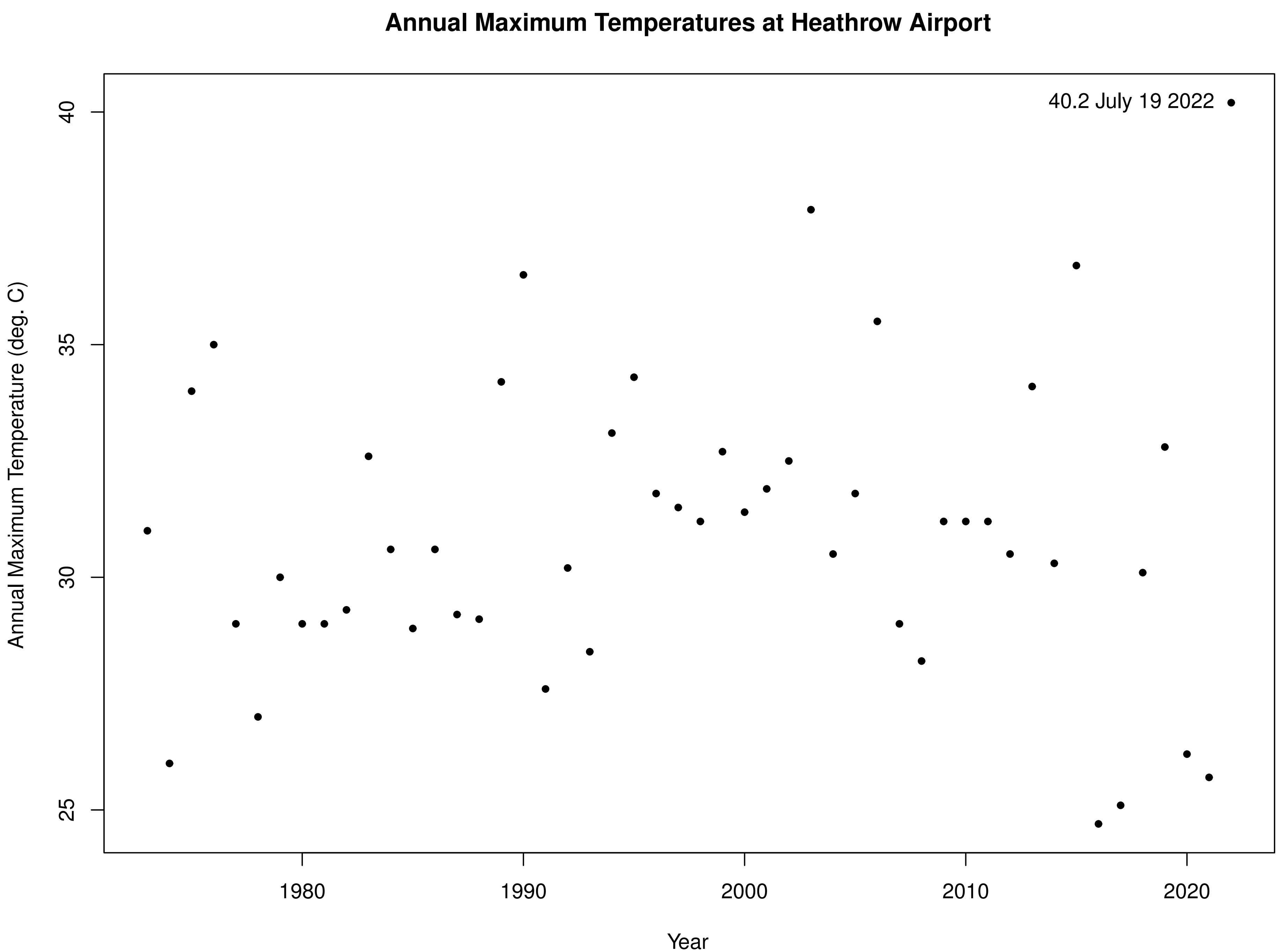
5. Second key idea: projections of future extreme event probabilities

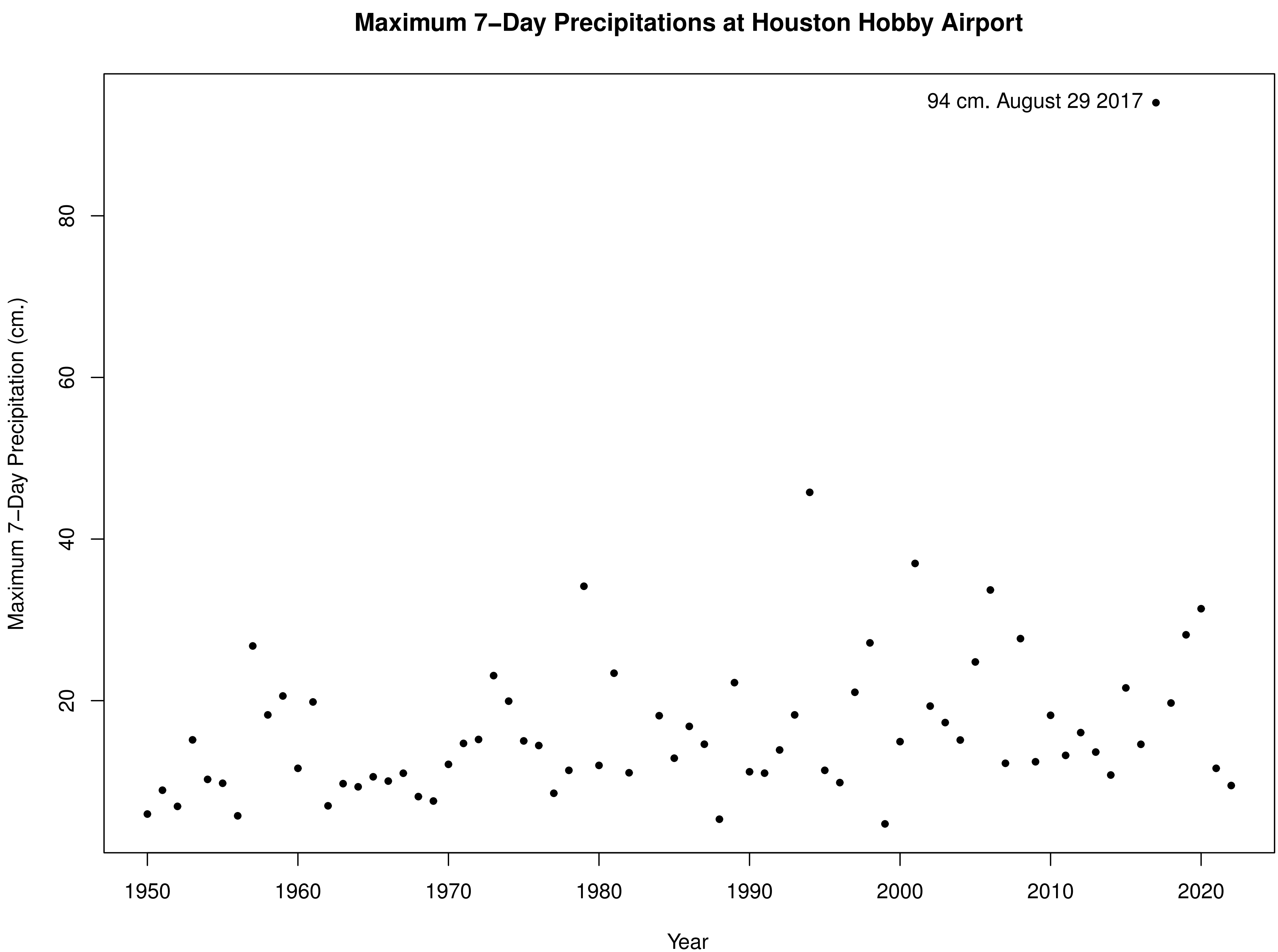
I. Introduction

I begin with three examples of datasets that contain extreme events



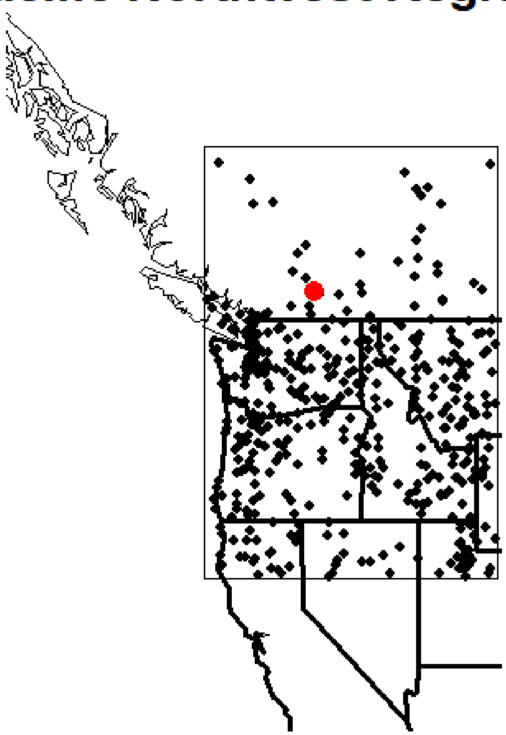
Annual Maximum Temperatures in Kelowna, BC
44.6 June 30 2021
Annual Maximum Temperature (deg. C)
32
34
36
38
40
42
44
1990
2000
2010
2020
Year

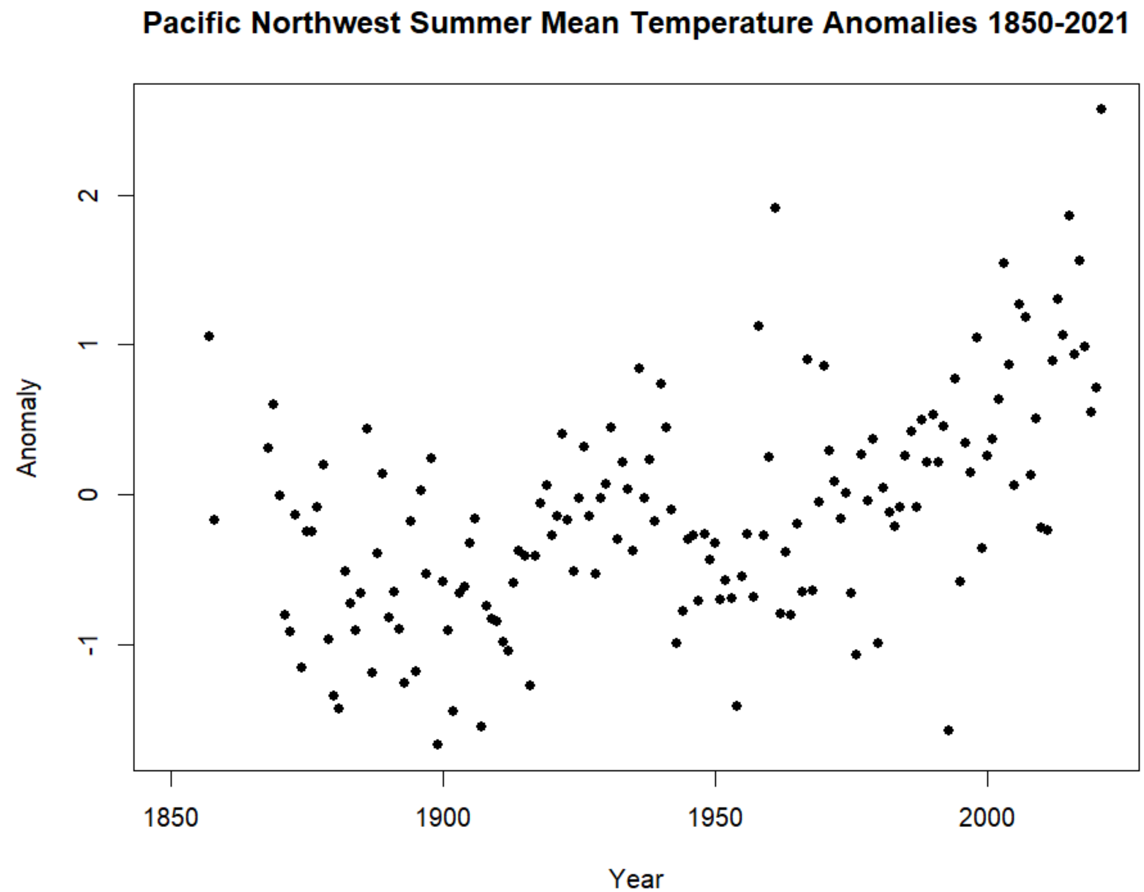

Annual Maximum Temperatures at Heathrow Airport
40.2 July 19 2022
Annual Maximum Temperature (deg. C)
25
30
35
40
1980
1990
2000
2010
2020
Year

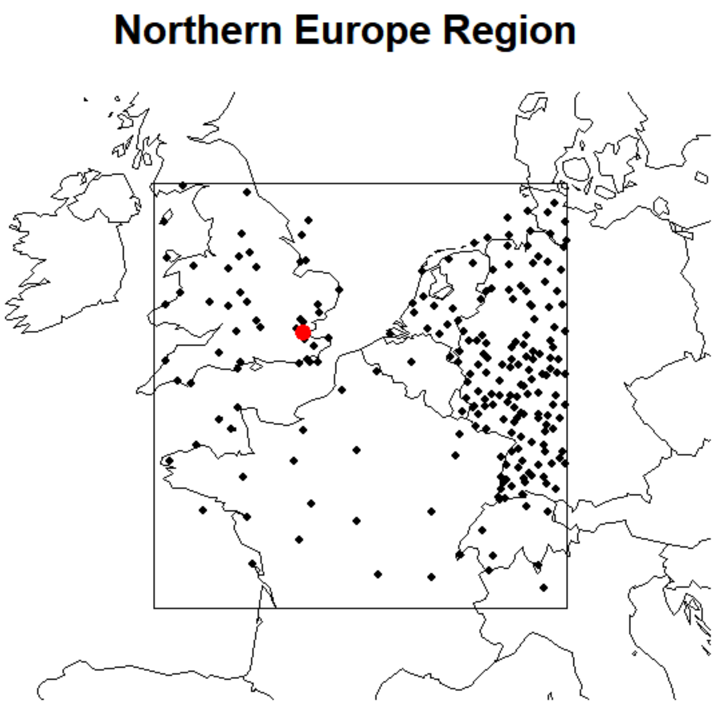

Maximum 7−Day Precipitations at Houston Hobby Airport
94 cm. August 29 2017
Maximum 7−Day Precipitation (cm.)
20
40
60
80
1950
1960
1970
1980
1990
2000
2010
2020
Year

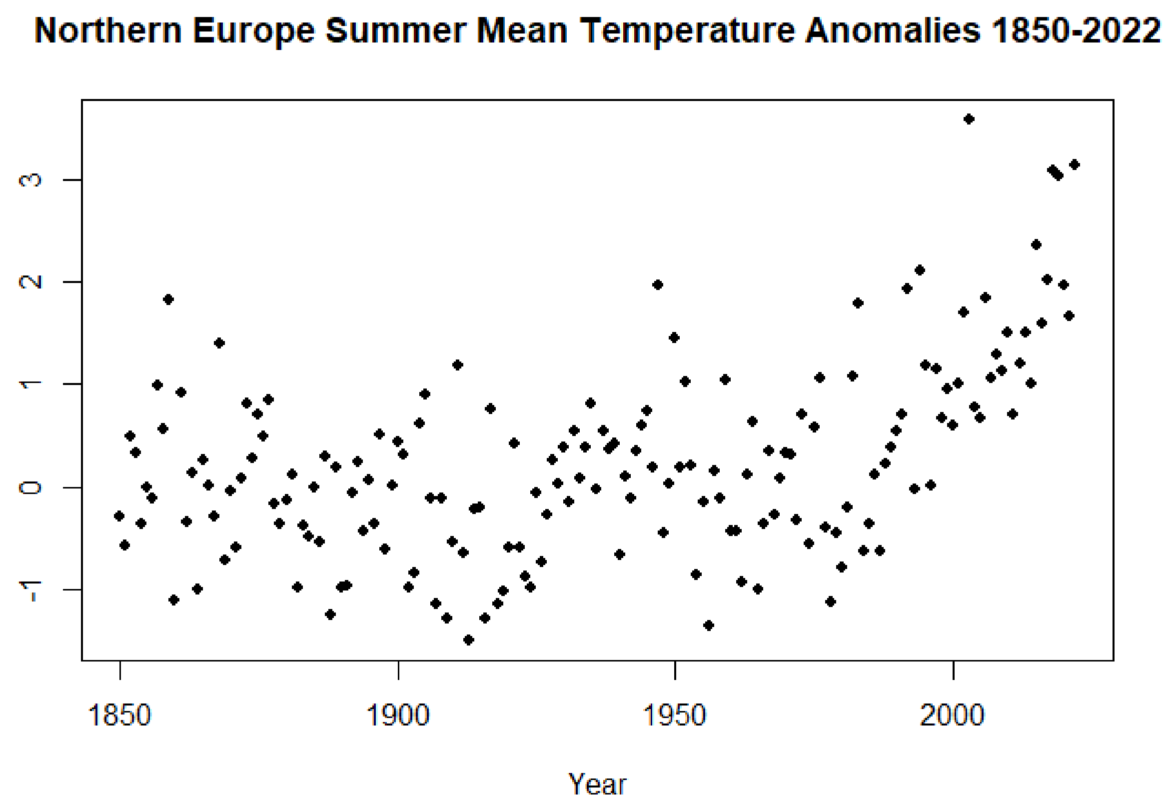
For each of these examples, I have collected weather data from multiple stations in the same region (from the Global Historical Climatological Network), and also calculated a *regional variable* that includes annual or seasonal maxima from spatially aggregated data (from the Climate Research Unit of the University of East Anglia)

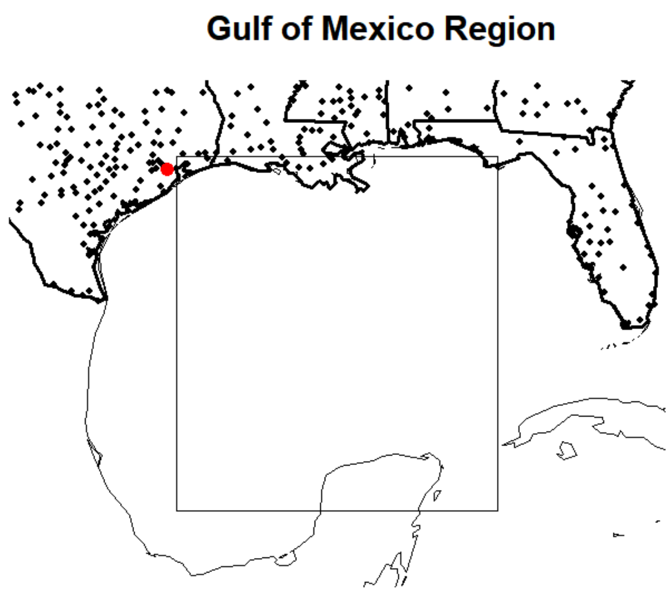
Pacific Northwest Region

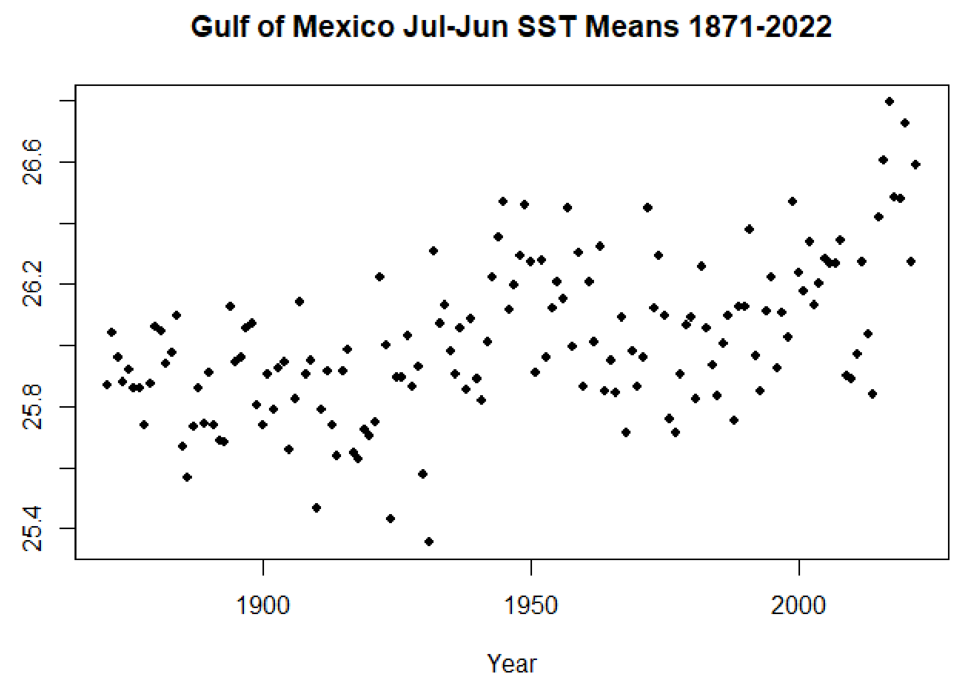




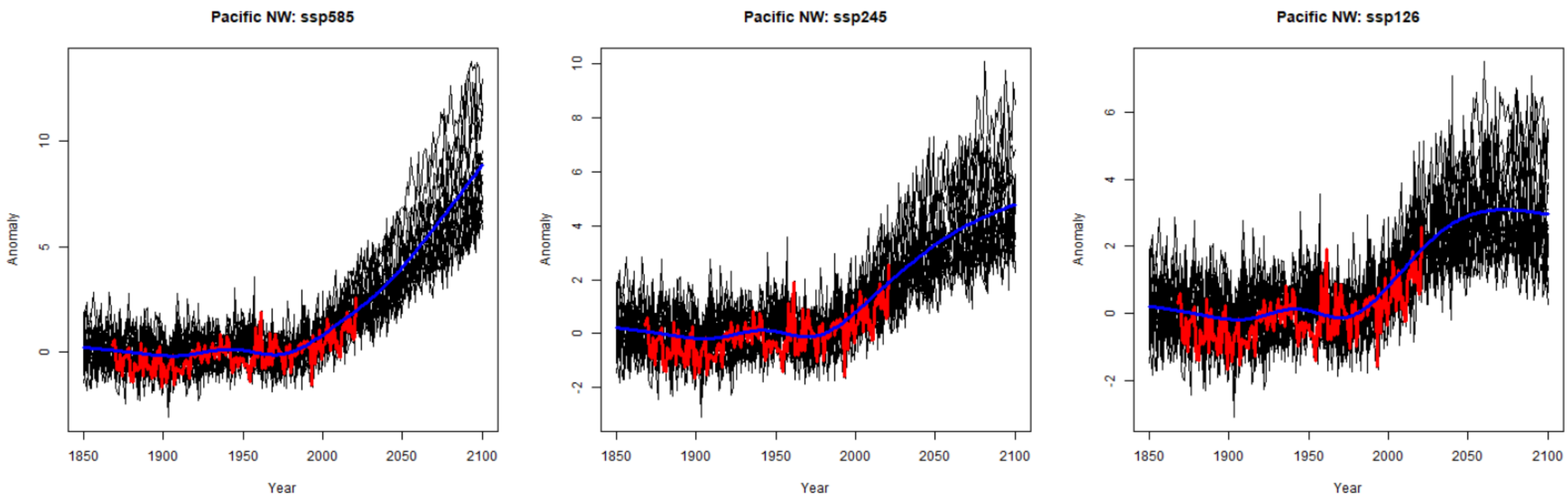

Northern Europe Region


Northern Europe Summer Mean Temperature Anomalies 1850-2022
3
2
1
0
-1
1850
1900
1950
2000
Year

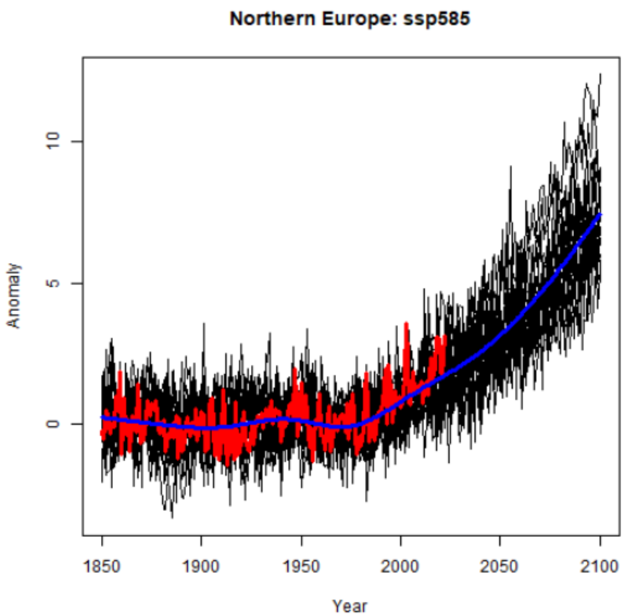

Gulf of Mexico Region

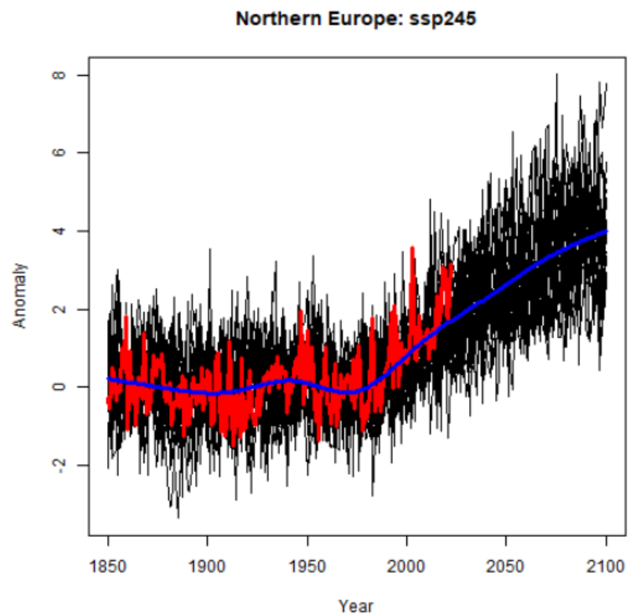

Gulf of Mexico Jul-Jun SST Means 1871-2022
26.6
26.2
25.8
25.4
1900
1950
2000
Year

I have also compiled 17 climate model datasets (from CMIP6) that correspond to the regional variables defined above



Pacific NW: ssp585
Pacific NW: ssp245
Pacific NW: ssp126
Anomaly
Year
1850
1900
1950
2000
2050
2100
0
5
10
-2
2
4
6
8


Northern Europe: ssp585
Anomaly
0
5
10
1850
1900
1950
2000
2050
2100
Year


Northern Europe: ssp245
Anomaly
-2
0
2
4
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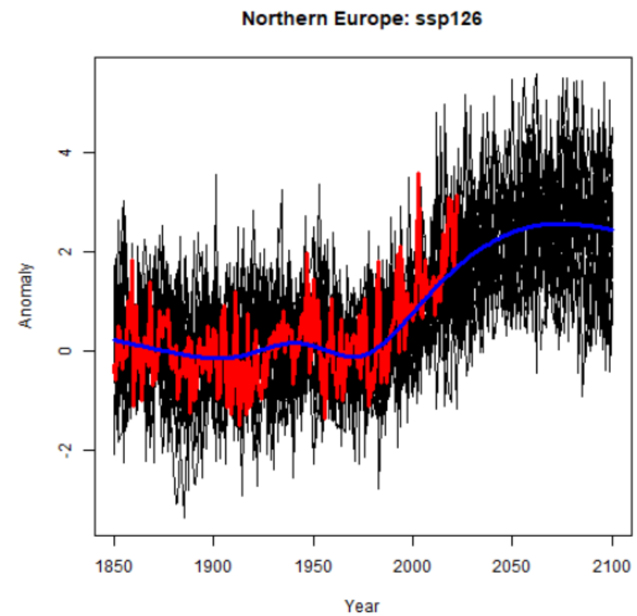
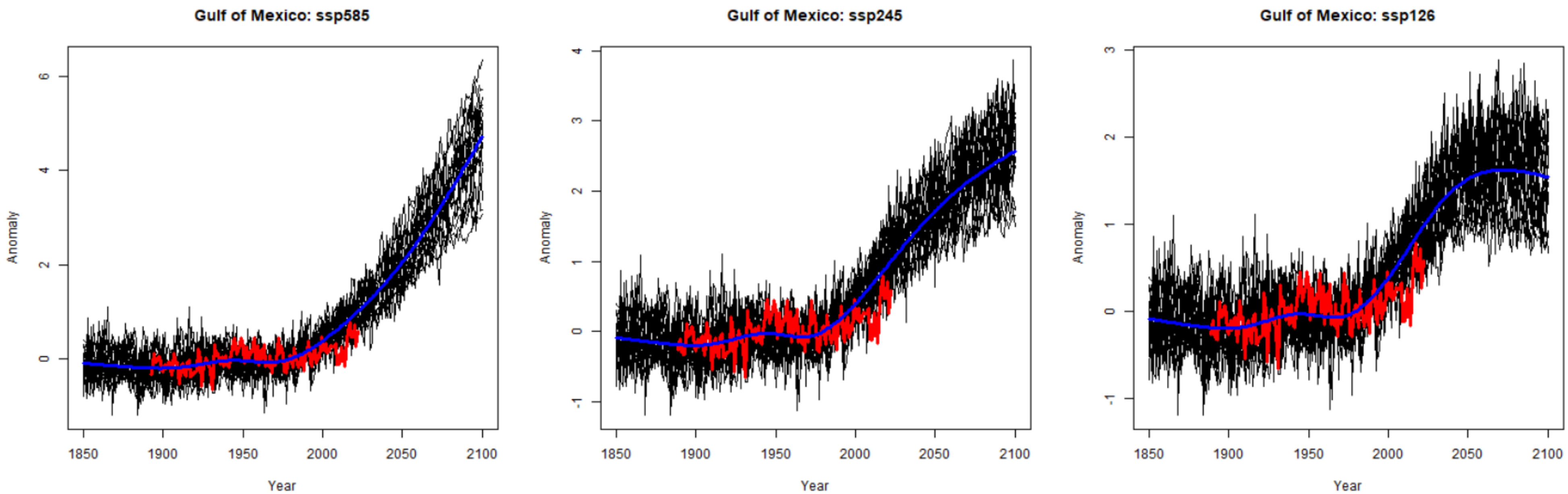
8
1850
1900
1950
2000
2050
2100
Year


Northern Europe: ssp126
Anomaly
-2
0
2
4
1850
1900
1950
2000
2050
2100
Year



Gulf of Mexico: ssp585
Gulf of Mexico: ssp245
Gulf of Mexico: ssp126
Anomaly
Year
1850
1900
1950
2000
2050
2100
0
2
4
6
-1
1
3

II. Statistical Analysis

IIa. Used the Generalized Extreme Value (GEV) for each station with regional variable as a covariate

IIb. Combine stations using a spatial model

IIc. Climate models to project the regional variable forwards and backwards in time

IId. “End to end” analysis to show how the extreme event probability changes corresponding to climate variation (including uncertainty bounds)

Aside: Background on Extreme Value Theory

Original papers: Fréchet (1927), Fisher and Tippett (1928), von Mises (1936), Gnedenko (1943)

Consider $X_1, X_2, \ldots$ IID random variables with distribution function F, $M_n = \max(X_1, \ldots, X_n)$. Find asymptotic distribution in form

$$\Pr\left\{\frac{M_n - b_n}{a_n} \le y\right\} \;=\; F^n(a_n y + b_n) \;\to\; G(y)$$

where a_n and b_n are normalizing constants and G is some limiting distribution.

Fisher-Tippett-Gnedenko theorem: if such a limit exists, $G(y)$ must be one of "three types" of probability distributions.

Von Mises showed they could be combined into a single distribution family which we nowadays call the *Generalized Extreme Value* (GEV) distribution.

GEV Distribution: Characterization

$$G(y; \mu, \psi, \xi) = \exp\left\{-\left(1+\xi\frac{y-\mu}{\psi}\right)_+^{-1/\xi}\right\}$$

valid whenever $1+\xi\frac{y-\mu}{\psi} > 0$.

- μ is the *location parameter* — determines center of distribution
- ψ is the *scale parameter* — how spread out the distribution is
- ξ is the *shape parameter*.
- When $\xi > 0$, $1 - G(y)$ ultimately behaves like $y^{-1/\xi}$ — Pareto tail — long-tailed. Also known as Fréchet distribution.
- When $\xi < 0$, the distribution has a finite endpoint at $\mu - \psi/\xi$ — short-tailed, equivalent to Weibull distribution
- The case $\xi = 0$ is interpreted as the limit $\xi \to 0$: in that case, $G(y) = \exp(-e^{-y})$, also known as the Gumbel distribution.
- Precipitation series usually follow a Pareto tail with $\xi \approx 0.1$ but there is a finite endpoint because of the *probable maximum precipitation*
- Temperature series usually follow a Weibull tail with $\xi \approx -0.2$ but this can create problems also

GEV Distribution: Estimation

$$G(y;\mu,\psi,\xi) \;=\; \exp\left\{-\left(1+\xi\frac{y-\mu}{\psi}\right)_+^{-1/\xi}\right\}$$

valid whenever $1+\xi\frac{y-\mu}{\psi}>0$.

- Sample $Y_1,\ldots,Y_N$, e.g. annual maximum temperatures at a specific location
- Maximum likelihood estimation (MLE): Define $g(y;\mu,\psi,\xi)=\frac{dG(y;\mu,\psi,\xi)}{dy}$, choose μ, ψ, ξ to minimize

$$\ell(\mu,\psi,\xi) \;=\; -\sum_{i=1}^{N}\log g(Y_i;\mu,\psi,\xi).$$

- First numerical optimization method proposed by Jenkinson (1969)
- Fisher information matrix calculated by Prescott and Walden (1980), more detailed maximum likelihood theory by Smith (1985)
- Many modern refinements, e.g. Zhang and Shaby (2021)
- Standard MLE theory holds when $\xi>-\frac{1}{2}$

Extensions of the GEV

- Alternative viewpoints, e.g. excesses over thresholds, Generalized Pareto distribution (Pickands 1975; Davison and Smith 1990; Coles 2001)

- Include covariates (e.g. Smith 1990 and many other references)

- Multivariate extremes (many references...)

- Spatial models: allow μ, ψ, ξ and any regression parameters to vary smoothly in space; fit a Gaussian process model (Coles and Casson 1999,...)

- Many forms of stochastic processes that directly allow for dependence among spatial locations, e.g. max-stable processes, max-id, scale mixtures of normals, etc.

IIa. GEV Analysis

$$G(y) = \Pr\{Y \le y\} = \exp\left\{-\left(1+\xi\frac{y-\mu}{\psi}\right)_{+}^{-1/\xi}\right\}$$

- Parameters μ, ψ, ξ depend on time and space
- Time dependence based on regional variable as a covariate
- Point of clarification: There is a debate in the literature about whether the analyzed data should include the extreme event of interest. The results I am showing here do *not* do this: the analyses for Kelowna, London and Houston are based on station data up to 2020, 2021 and 2016 respectively.

Covariate Models

(Risser and Wehner 2017, Russell et al. 2020)

$$\begin{aligned} \mu_{s,t} &= \theta_{s,1} + \theta_{s,4} X_t, \\ \log \psi_{s,t} &= \theta_{s,2} + \theta_{s,5} X_t, \\ \xi_{s,t} &= \theta_{s,3}, \end{aligned}$$

Define a parameter vector $\mathbf{\Theta}_s = \begin{pmatrix} \theta_{s,1} & \dots & \theta_{s,5} \end{pmatrix}$ at each site s; a 5-dimensional parameter vector for each site s.

Extension: $\log \left\{ \frac{1+\xi_{s,t}}{1-\xi_{s,t}} \right\} = \theta_{s,3} + \theta_{s,6} X_t$ (6-parameter model), also combined into $\mathbf{\Theta}_s$

IIb. Spatial Extremes Analysis

Objective: Come up with a model for interpolating the GEV distributions between stations, and also improving the analysis at individual stations by “borrowing strength” across stations.

- Latent process approach: Russell, Risser, Smith and Kunkel (2020)

- Idea is to “combine strength” across different stations

- Fit a spatial model to all the stations, then project backwards to specific locations (including the stations)

- Several other approaches, see in particular Zhang, Risser, Wehner and O’Brien (forthcoming)

Concept of Approach

- True (latent) process $\mathbf{\Theta}$ (KN-dimensional, K=5 or 6)
- Estimated process $\hat{\mathbf{\Theta}}$ (GEV estimates at each site)
- Assume $(\hat{\mathbf{\Theta}} \mid \mathbf{\Theta}) \ \sim \ \mathcal{N}_{KN}(\mathbf{\Theta}, W)$
- Spatial model $(\mathbf{\Theta} \ \sim \ \mathcal{N}_{KN}(\boldsymbol{\mu} \otimes I_N, V(\phi)))$
- $\hat{\mathbf{\Theta}} \ \sim \ \mathcal{N}_{KN}(\boldsymbol{\mu} \otimes I_N, V(\phi) + W)$
- Estimate W empirically, $\boldsymbol{\mu}$ and ϕ by MLE
- Hence generate $\mathbf{\Theta} \mid \hat{\mathbf{\Theta}}$
- Model for $V(\phi)$: *co-regionalization* (Wackernagel 2003, Finley et al. 2008, etc.)

Kelowna, B.C. (Single Station Approach)

5-Par Model:

Parameter	Estimate	SE	t-val	p-val
θ_1	34.8265	0.2511	138.7138	0.0000
θ_2	0.0703	0.1812	0.3882	0.6979
θ_3	-0.3709	0.3533	-1.0497	0.2939
θ_4	1.8317	0.2708	6.7642	0.0000
θ_5	-0.0958	0.3372	-0.2841	0.7763

MLE probability of exceeding 44.6°C in 2021, given X_{2021}: 0.

Bayesian posterior mean: 0.012 (1-in-83-year event, even *given* the high regional temperature)

6-Par Model:

Parameter	Estimate	SE	t-val	p-val
θ_1	34.8386	0.2809	124.0051	0.0000
θ_2	0.1397	0.1866	0.7486	0.4541
θ_3	-0.9475	0.4582	-2.0679	0.0386
θ_4	1.8494	0.2686	6.8861	0.0000
θ_5	-0.2301	0.2755	-0.8352	0.4036
θ_6	1.1113	0.7217	1.5399	0.1236

MLE probability for 2021 is 0.072, Bayesian 0.076 (1-in-13-year)

Results: Kelowna (6-Par Spatial Model)

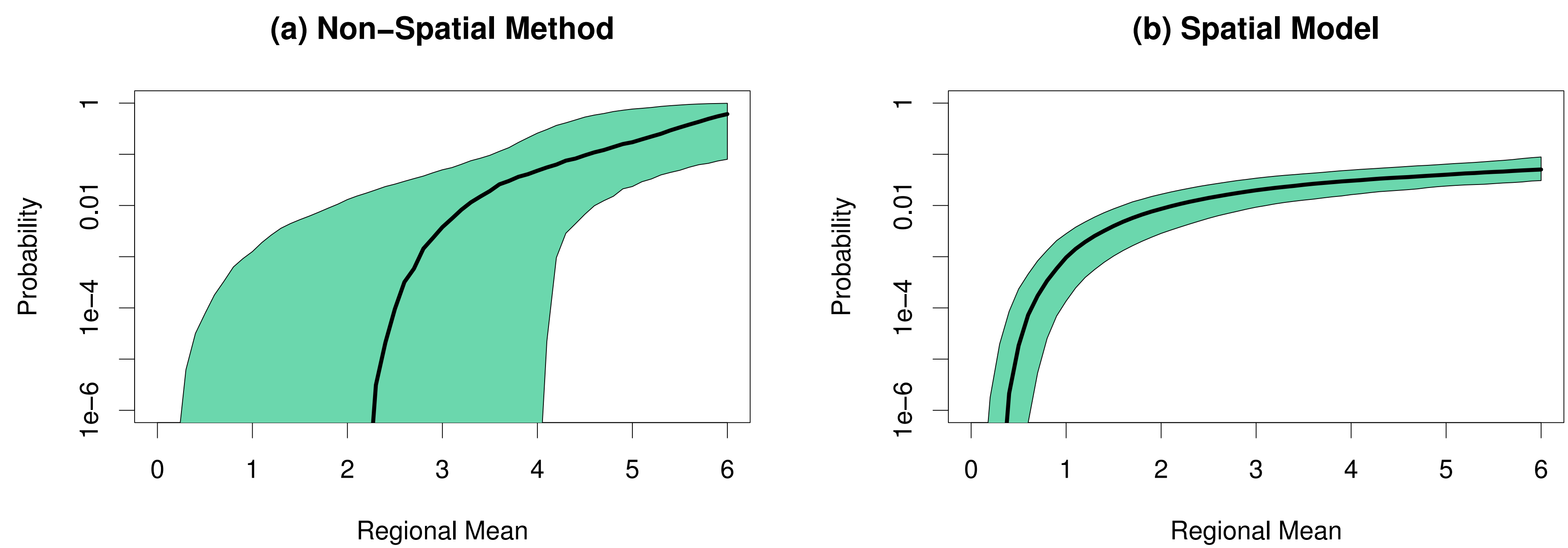
MLE Analysis

Parameter	Estimate	SE	t-val	p-val
θ_1	34.8386	0.2809	124.0051	0.0000
θ_2	0.1397	0.1866	0.7486	0.4541
θ_3	-0.9475	0.4582	-2.0679	0.0386
θ_4	1.8494	0.2686	6.8861	0.0000
θ_5	-0.2301	0.2755	-0.8352	0.4036
θ_6	1.1113	0.7217	1.5399	0.1236

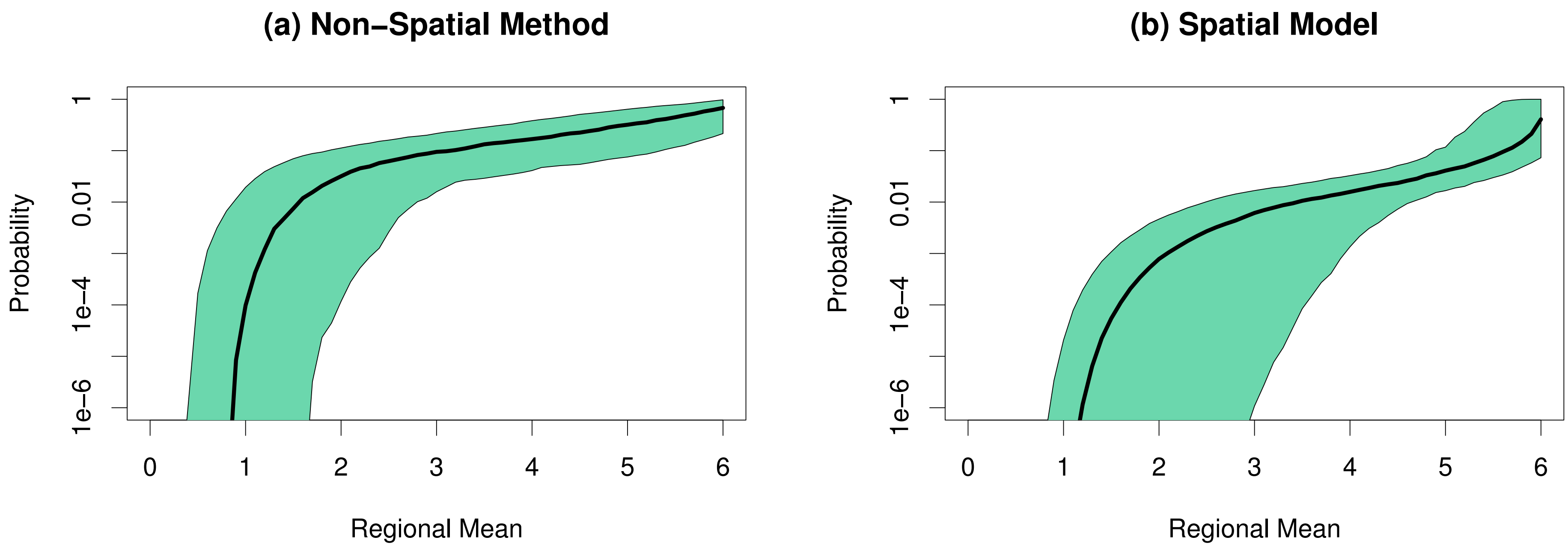
Spatial Analysis

Parameter	Estimate	SE	t-val	p-val
θ_1	34.8437	0.1767	197.2273	0.0000
θ_2	0.1099	0.0808	1.3597	0.1739
θ_3	-0.5908	0.1272	-4.6438	0.0000
θ_4	1.7402	0.1530	11.3750	0.0000
θ_5	-0.3748	0.1219	-3.0754	0.0021
θ_6	0.4290	0.2025	2.1185	0.0341

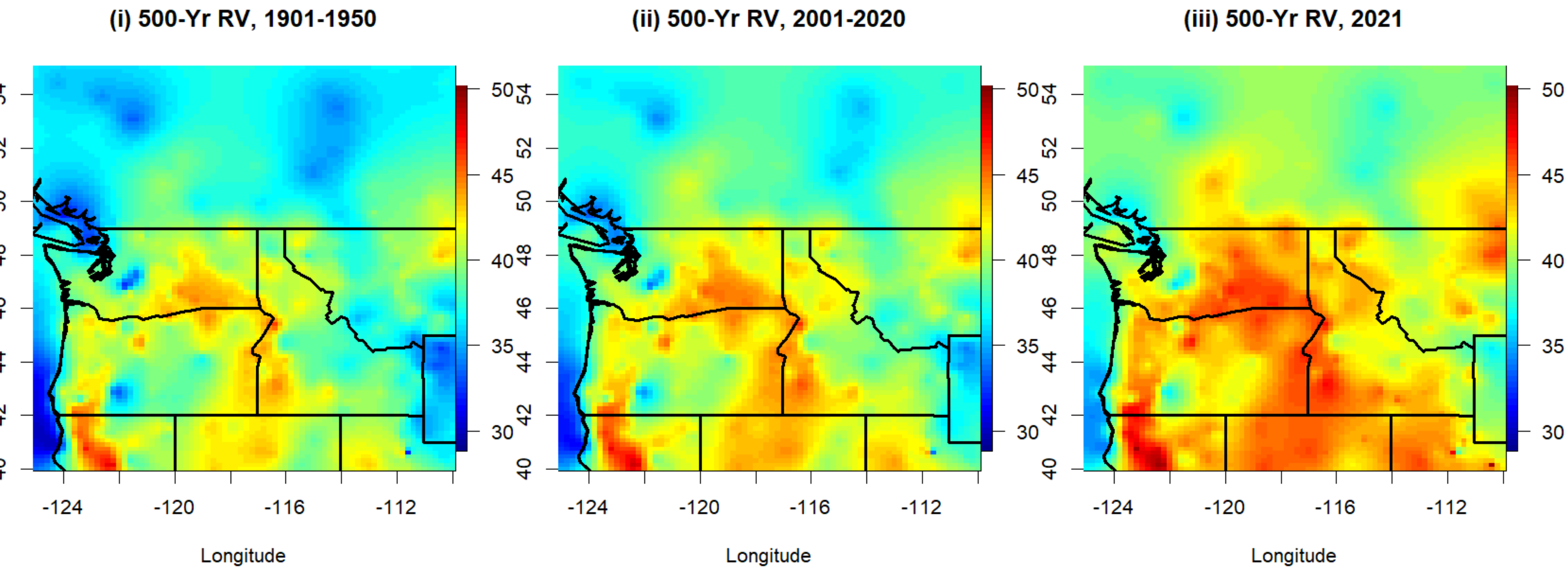
Estimates and 66% Credible Intervals for Mean Exceedance Probability: Comox, B.C.



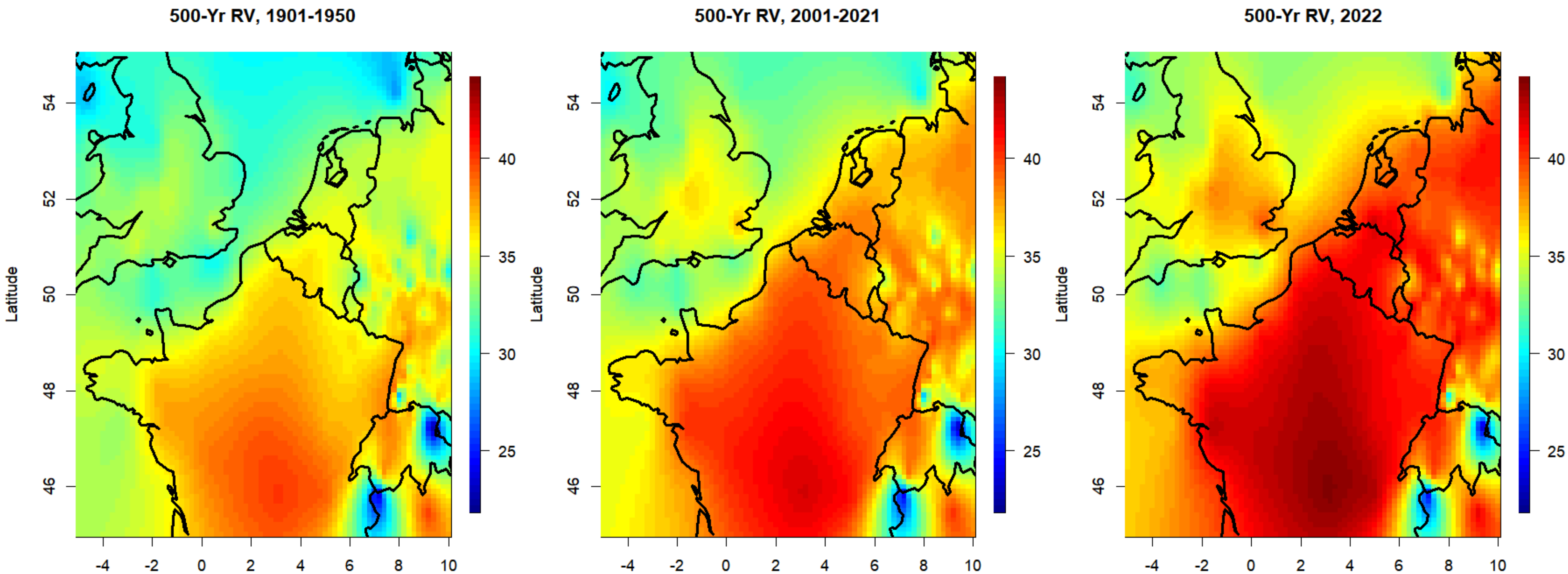
Estimates and 66% Credible Intervals for Mean Exceedance Probability: Kelowna (with monotonicity constraint)



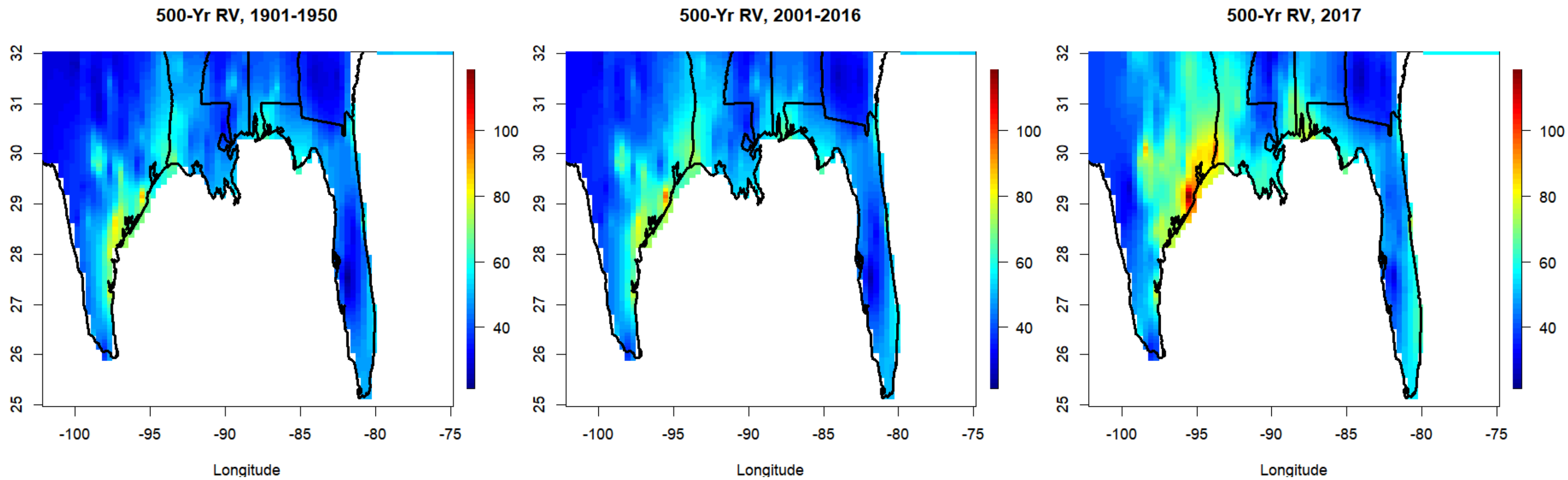
PNW: 500-year return values for (i), (ii), (iii)



NEU: 500-year return values for (i), (ii), (iii)



GOM: 500-year return values for (i), (ii), (iii)



Houston, we have a problem

Looking at the Probabilities of Individual Events

Conditional probabilities of exceeding 2021 temp in PNW:

	(i) 1901–1950	(ii) 2001–2020	(iii) 2021
Kelowna (44.6°C)	3×10^{-12}	8.6×10^{-6}	0.0061
All Canadian stations	0.0081	0.0185	0.067

Conditional probabilities of exceeding 2022 temp in UK:

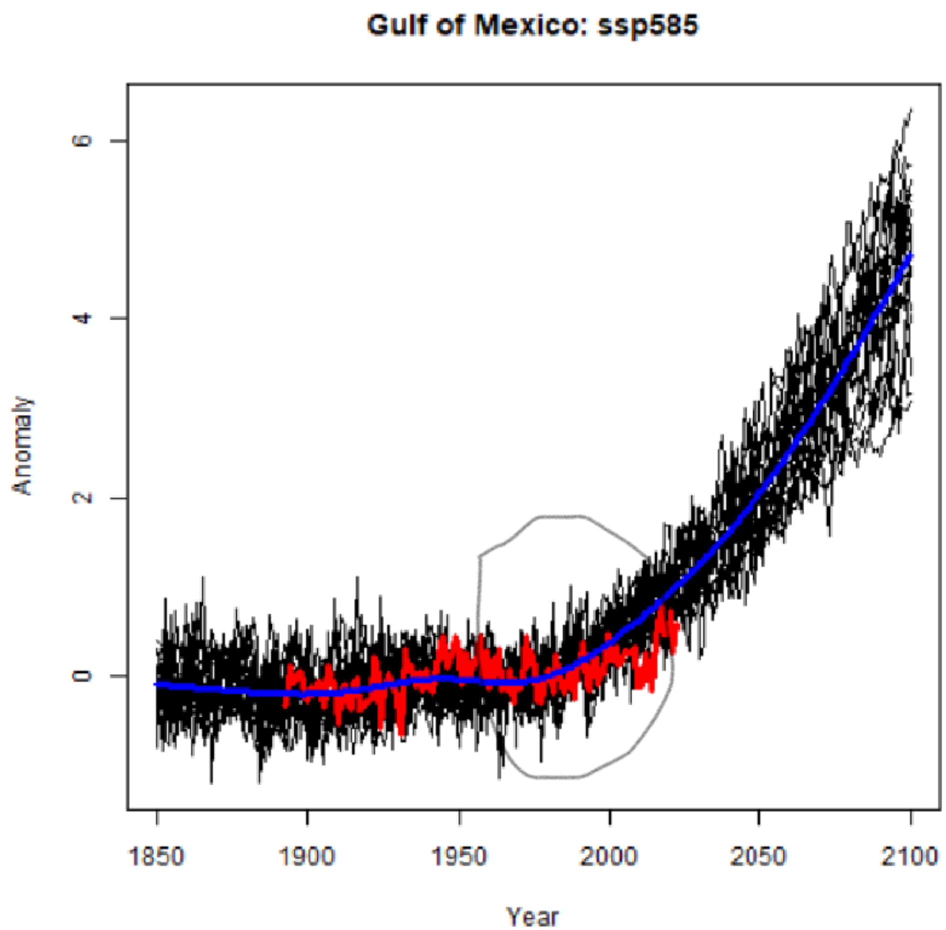
	(i) 1901–1950	(ii) 2001–2021	(iii) 2022
Heathrow	0	3.1×10^{-5}	0.017
All U.K. stations	0.0081	0.0319	0.095

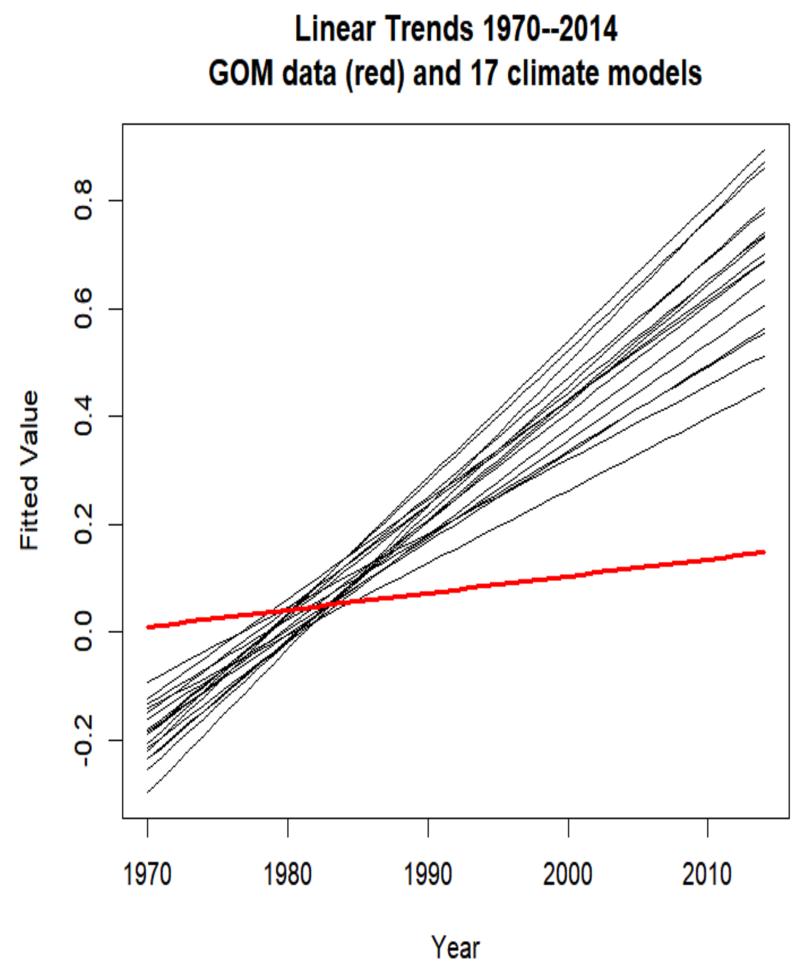
Conditional probabilities of exceeding 2017 precip in Houston:

	(i) 1901–1950	(ii) 2001–2016	(iii) 2017
Houston Hobby	4.7×10^{-5}	0.00014	0.0023
All stations $>$ 70 cm	0.00017	0.00030	0.0023

Still haven't introduced climate models into the discussion

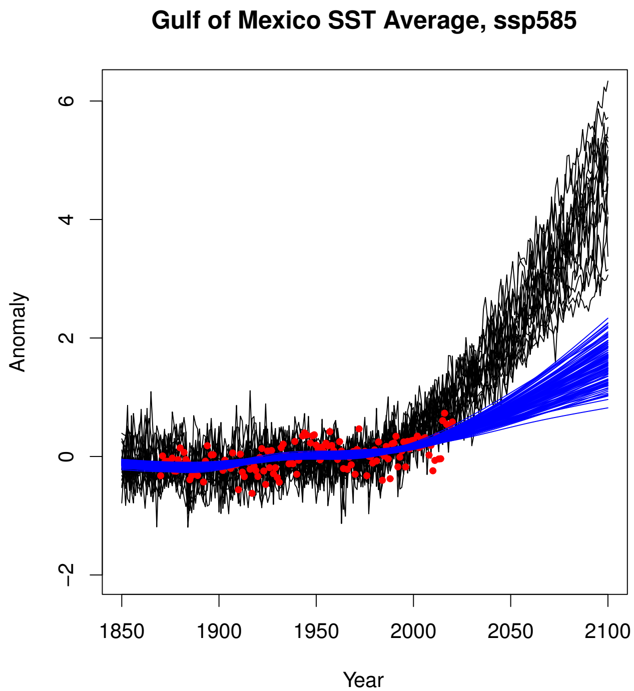
IIc: Projecting the Distribution of the Regional Variable Forwards and Backwards in Time

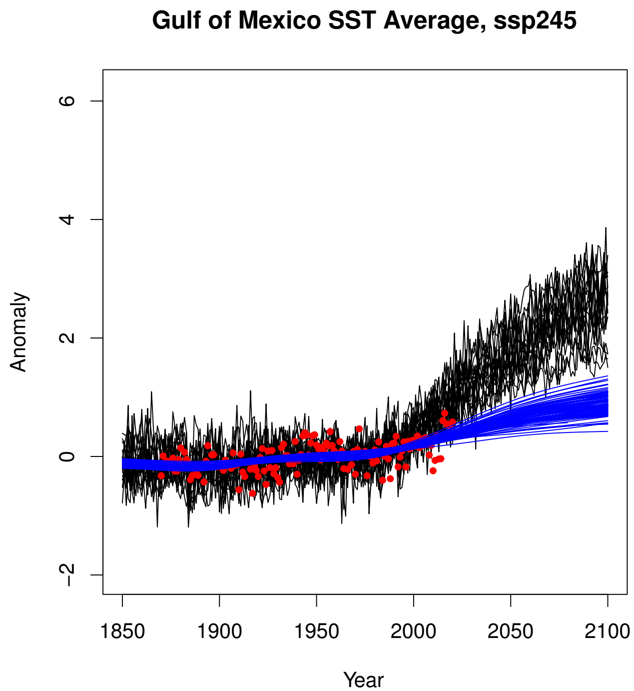


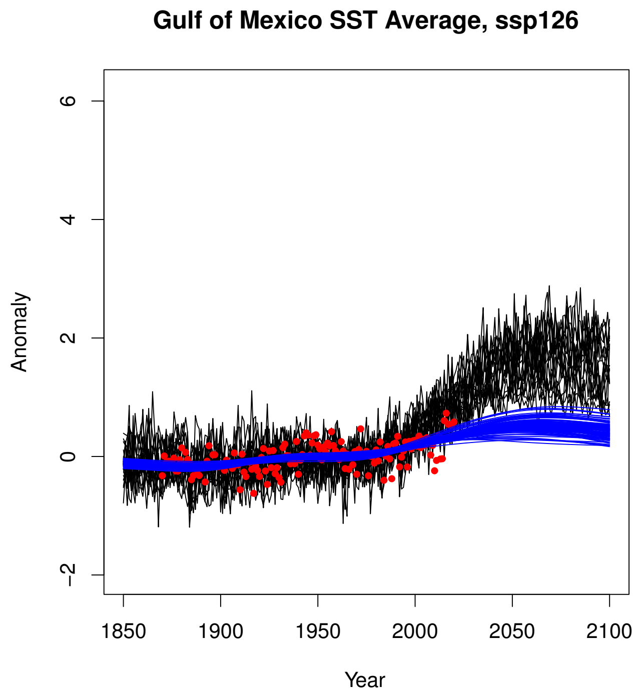


- Obvious method: regress observed regional value on 17 climate models, then use standard prediction theory
  - Objection: ignores variability in the covariates (climate model)
- To accommodate this feature, we need a model for the joint error distribution of 17 climate models. They are not independent!
- Typical solution: use principal components (empirical orthogonal functions), but it's not clear how to accommodate variability in the PCs (side note: Katzfuss, Hammerling and Smith (2017, GRL) proposed a Bayesian solution to detection and attribution, but did not resolve this question)
- Alternative: factor analysis (FA) instead of PCs
- FA models are based on unobserved latent components, easy to implement via Gibbs sampling (don't need Metropolis)
- But..... still susceptible to overfitting, possible lack of proprietary of posterior distribution
- I have avoided these issues by using a "shrinkage prior" formulation of Bhattacharya and Dunson (2011), allows arbitrarily many factors (I actually used 2)

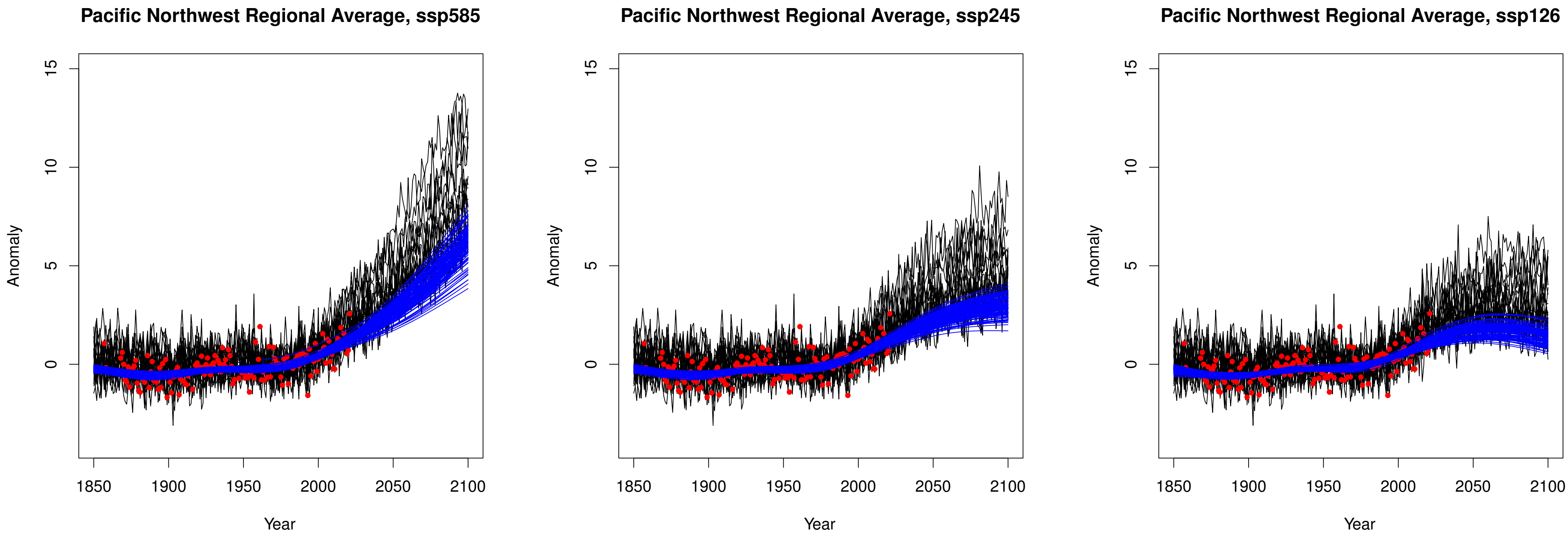
Regional Variable Projections: Gulf of Mexico



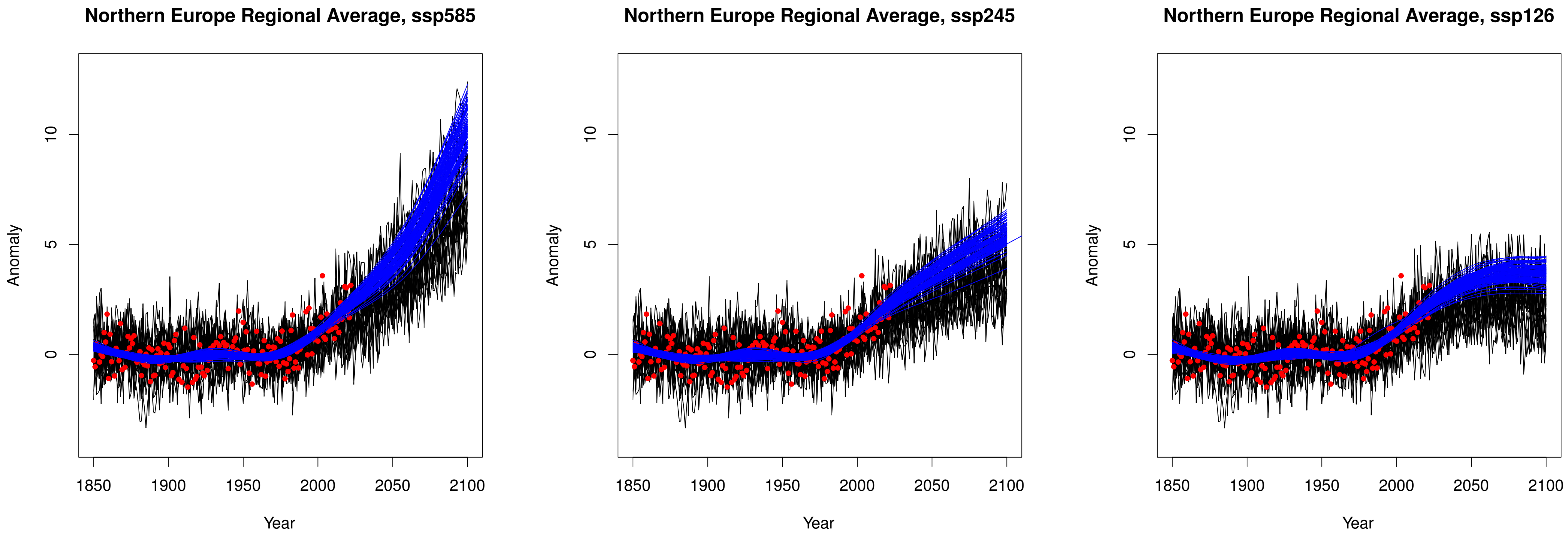




Regional Variable Projections: Pacific Northwest



Regional Variable Projections: Northern Europe



IId: End to End Analysis

- Generate Monte Carlo sample for regional variable condition on climate models

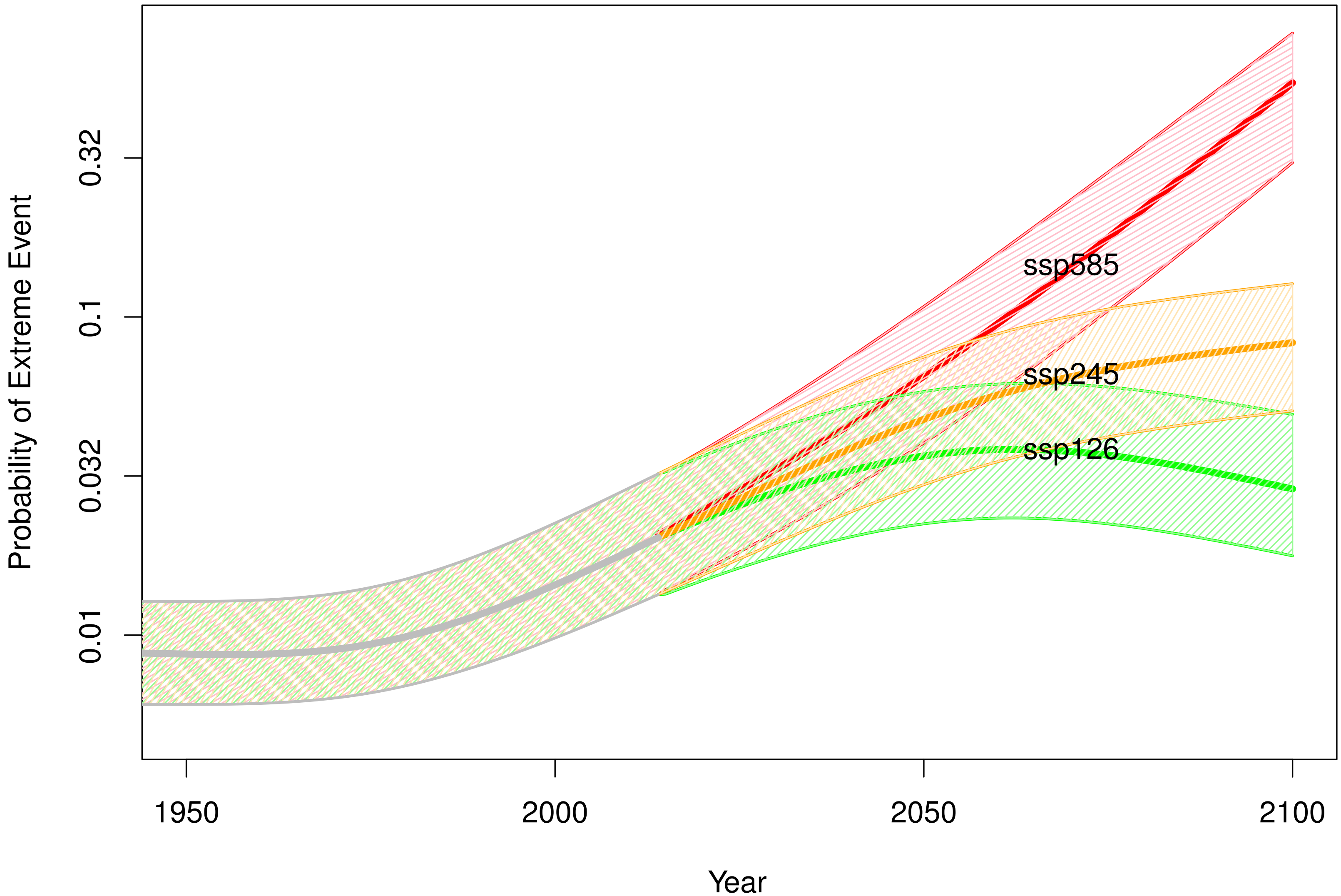
- Conditional on the regional variable, use the spatial GEV model to simulate values of the exceedance probabilities

- Compute 66% prediction intervals ("likely" in IPCC terminology)

- Plot the results

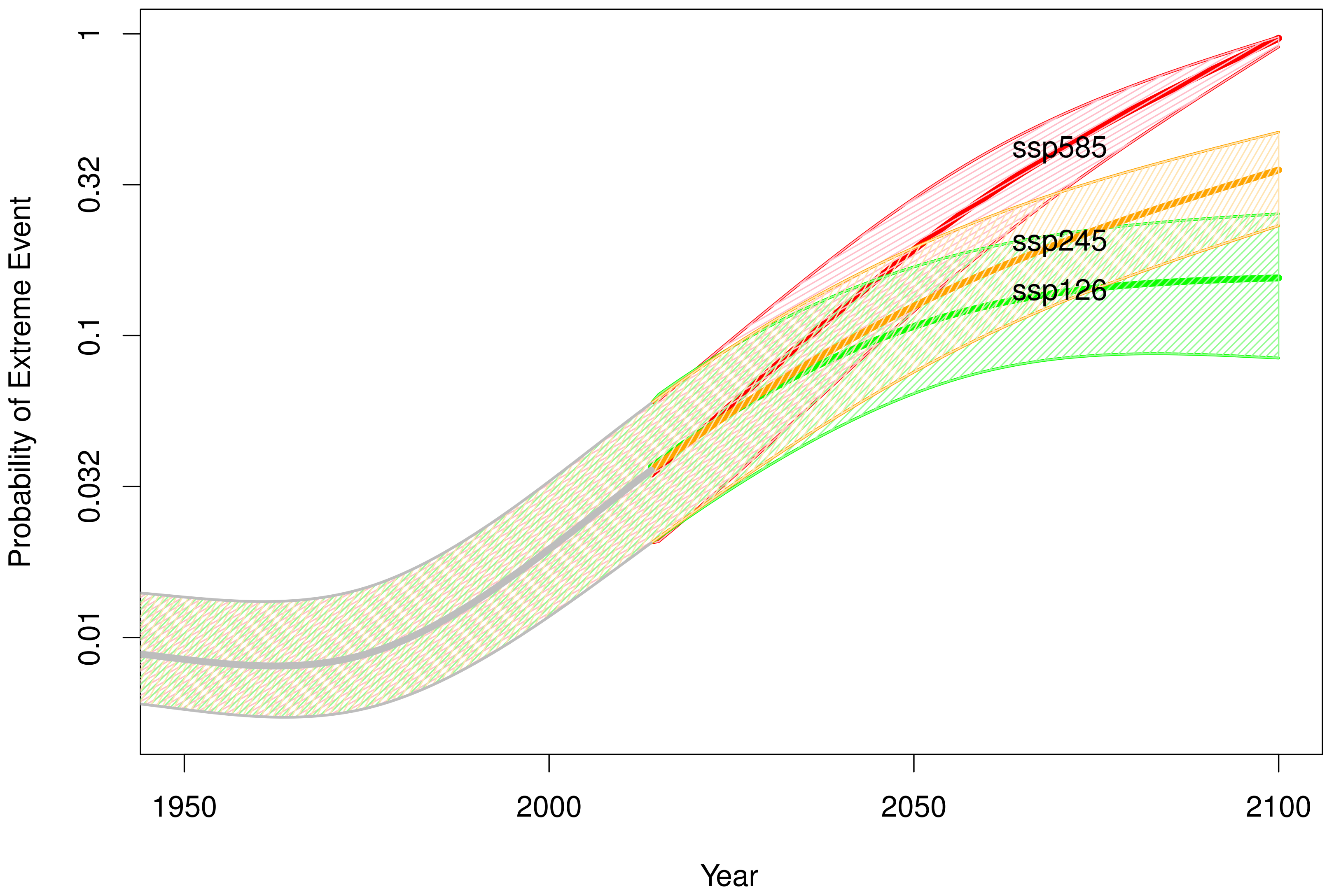
End To End Analysis: Mean Probability of Exceeding 2021 Value for All Stations in Canada

Mean probability over 1850–1949: 0.008; for 2023: 0.025;
for 2080: (0.035, 0.072, 0.22) under three scenarios; for 2100: (0.029, 0.083, 0.54)



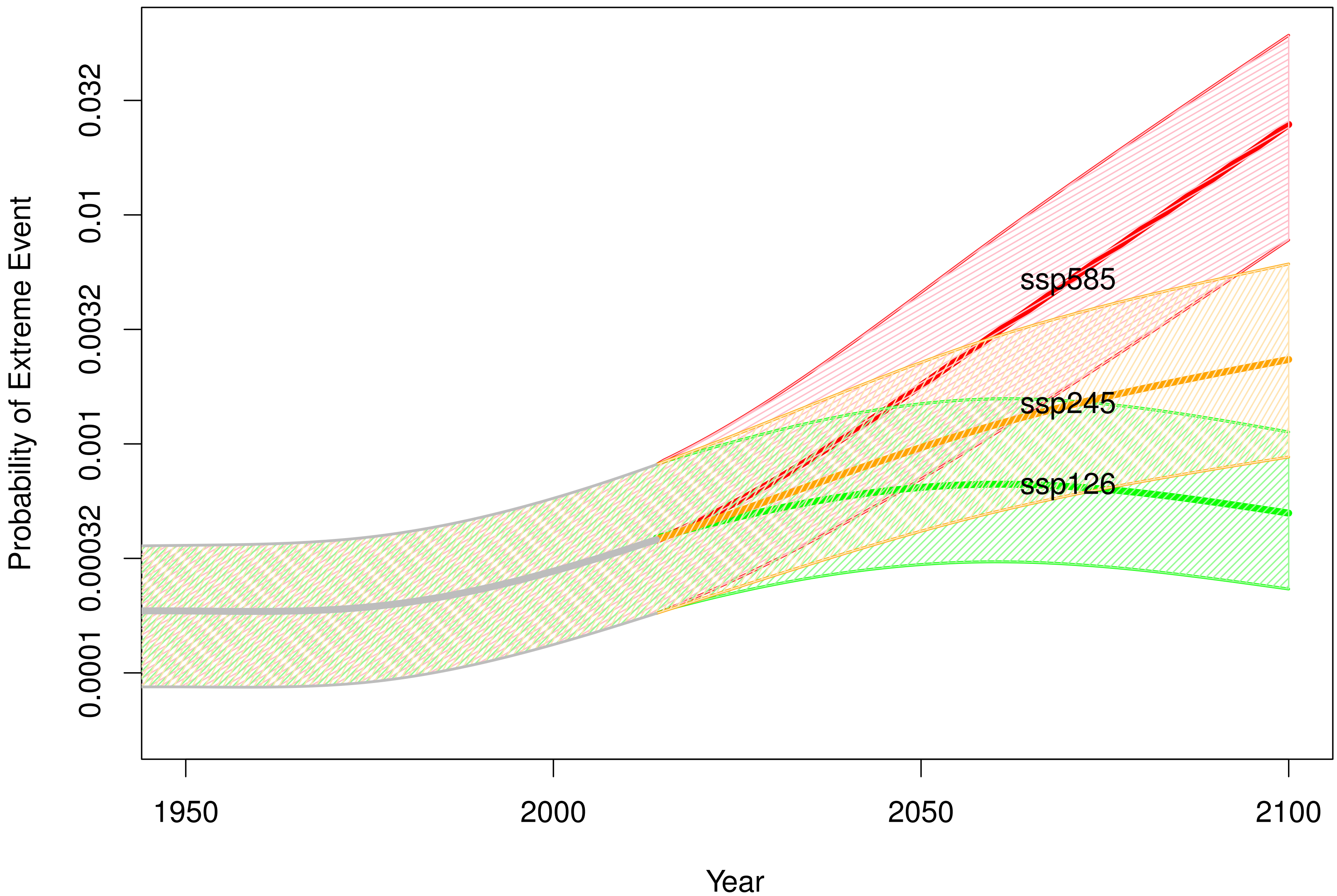
End To End Analysis: Mean Probability of Exceeding 2022 Value for All Stations in U.K.

Mean probability over 1850–1949: 0.008; for 2023: 0.052;
for 2080: (0.15, 0.25, 0.56) under three scenarios; for 2100: (0.16, 0.35, 0.97)



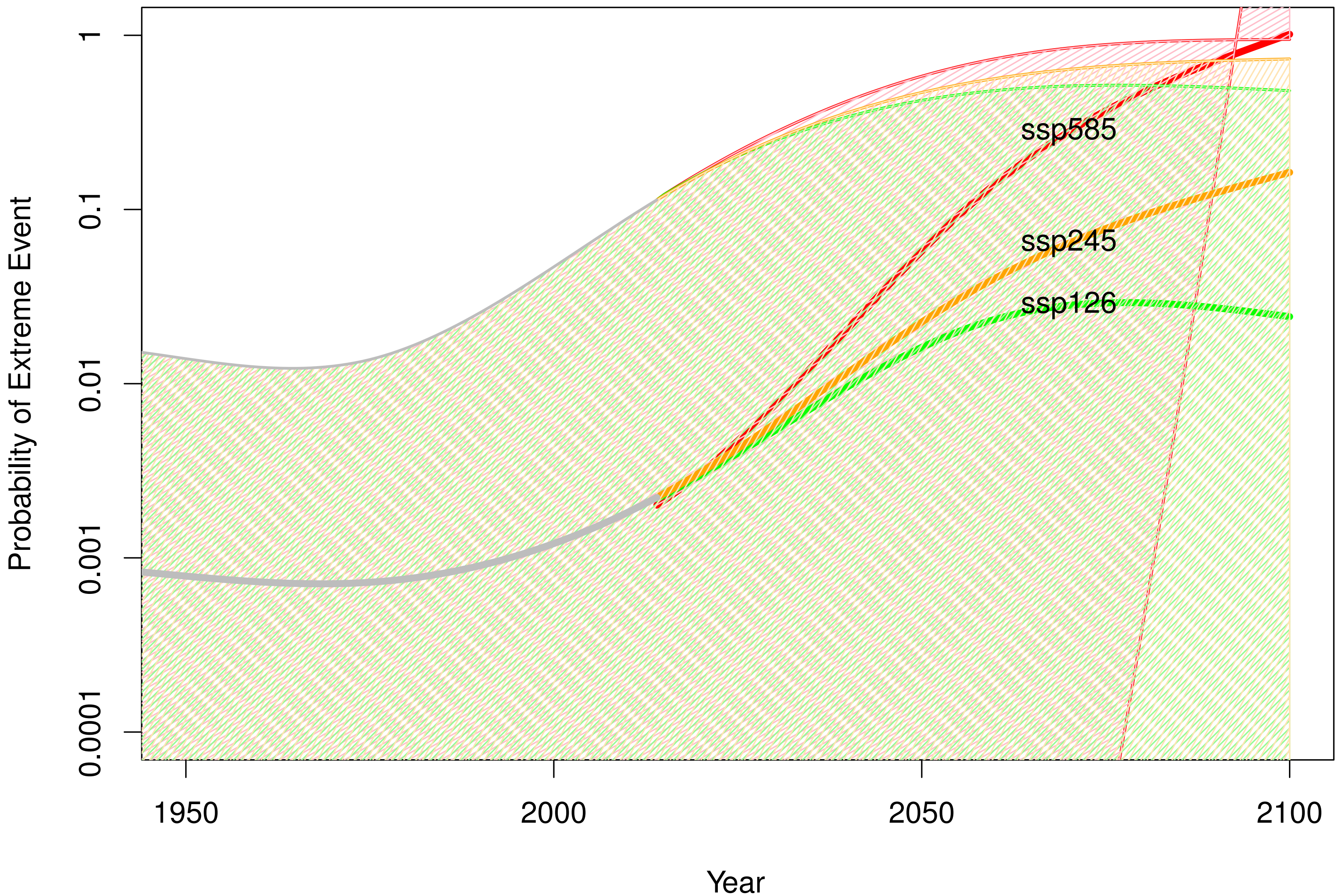
End To End Analysis: Mean Probability of Exceeding 2017 Value for 8 Stations near Houston

Mean probability over 1850–1949: 0.00015; for 2023: 0.00048;
for 2080: (0.00061, 0.0017, 0.0086) under three scenarios;
for 2100: (0.0005, 0.0023, 0.024)



End To End Analysis: Probability of Annual Maximum Exceeding 40 °C in Cardiff (unedited figure)

Mean probability over 1850–1949: 0.0009; for 2023: 0.0037;
for 2080: (0.029, 0.092, 0.47) under three scenarios;
for 2100: (0.024, 0.16, 1)



III: Conclusions and Policy Implications

- We have only considered three scenarios for the future, and there are many others, but the analysis demonstrates that there is a *huge* difference among the scenarios for projected probabilities of future extreme events

- Calculation of confidence/prediction/credible intervals is a key point of this analysis. We need to *quantify uncertainty*

- The important caveat: this analysis still relies on statistical assumptions that are not directly verifiable. We need a range of alternative approaches in order to demonstrate that the qualitative conclusions are not dependent on one particular method of analysis.

Slides and datasets: http://rls.sites.oasis.unc.edu/ClimExt/intro.html