DISCUSSION
of talks by Ben Lee, Noel Cressie and Raphaël Huser

Richard L. Smith

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Extreme Value Theory at a turning point

- Most of the "big names" from the 1980s are now gone
- But beyond that, I think the emphasis of the whole subject has changed
  - Then: the subject was essentially defined by the limit theorems, and trying to find statistical methods that preserved limiting relationships
  - Now: much more about finding computational techniques to solve practical problems

A bit of history

- Beginning: the “Three Types Theorem”
  - Fisher & Tippett (1928); Gnedenko (1943); De Haan (1970)
  - Von Mises (1936) first to formulate as a single family (GEV)
- Early statistical methods (Gumbel, Jenkinson, ...) focused on estimating parameters within these families

Limit theorems for other extremal problems

- Peaks over thresholds: the GPD
  - Balkema & de Haan (1974), Pickands (1975)

- Multivariate extremes
  - Early examples: Gumbel, Tiago de Oliveira,...
  - Characterization theorems: De Haan & Resnick, Pickands, Deheuvels
  - Statistical methods: Tawn, Coles, Ledford, Heffernan, Wadsworth, ... , Rootzén, Segers, Fougères, Cooley,...
  - The emphasis in these papers was still on statistical methods that were consistent with classical limit theorems and properties such a multivariate regular variation (later: hidden regular variation)

- Extremes in dependent sequences
  - Probability theory: Leadbetter, Lindgren & Rootzén (1983); Hüsler; Hsing; *extremal index* key parameter
  - Applications to specific models: Rootzén, Perfekt, Yun..
  - Estimation: Nandagopalan (1990), Smith & Weissman (1994), Ferro & Segers (2003), Holesovsky & Fusek (2020)
- Spatial extremes
  - Max-stable processes: De Haan (1984), Giné, Hahn & Vatan (1990)
  - Early attempts at statistical estimation: Smith (1990), Coles (1993), Schlather (2002)
  - The floodgates open: Kabluchko, Schlather & de Haan (2009), Padoan, Ribatet & Sisson (2010)
  - Many, many extensions...
  - 2024 preprint by Huser, Opitz & Wadsworth, "... Time to move away from max-stable processes"

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- Problems with older approaches to spatial extremes (e.g. max-stable processes): difficulty in computing likelihood function, lack of scalability, transition between dependence classes
- Scale-mixture approach (Huser & Wadsworth): $X(s,t) = R(t)^{\delta} W(s)^{1-\delta}$
- This paper: $X(s,t) = R(t)^{\alpha} g(Z(s))$ with $R(t)$ random
- In either version, $X(s,t)$ is not the observed process: match to $Y(s,t)$ by copula
- Numerous methods of estimation, e.g. Vecchia, SPQR, Neural Bayes...

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- Numerous methods of estimation, e.g. Vecchia, SPQR, Neural Bayes...
- A concern: every time these models run into a difficulty, they make a small tweak to the model and declare the problem solved. Will they every reach a stable point that it's realistic for climatologists to actually learn these methods?

Cressie and Cao

- Reconstructing the field of CO_2: important for assessing climate change and possible mitigation strategies
- OCO-2 satellite: recent technology from NASA, finest resolution of any dataset but significant gaps in coverage
- New idea: moving window approach to reconstruct missing data
- Some themes reminiscent of classical time series analysis:
  - Decomposition into long-term trend, monthly trend and seasonal variation
  - Extremes identified from anomalies (permutation test)
  - Not "extreme value theory" as such, but potentially valuable in detecting changes in the spatio-temporal pattern
- Ultimate objective is source apportionment (inverse problem)

Huser and co-authors

- Neural Bayes estimators: minimize $\frac{1}{K}\sum_{k=1}^{K} L(\theta^{(k)}, \widehat{\theta}_\gamma(Y^{(k)}))$
- Training is slow, estimation very fast ("amortization")
- Examples:
  - Red Sea SSTs
  - Global SSTs (non-stationary)
  - $PM_{2.5}$ in Saudi Arabia (alarmingly high)
  - UK precipitation
  - Arctic sea ice
  - All familiar problems where spatial extremes are critical
- Analogy with ABC methods
- R package
- Theoretical properties (Röder et al.): decomposition of error into three components, uses Cybenko (1989) theorem

Conclusions

- These two conferences (EVA2025 and CASE) have shown many new directions for extreme value theory

- Old obsession with limit theorems is gone, emphasis very much on scalable methods for large datasets

- Also notable: rapid growth in number of researchers into these methods

- There is still work to be done on communicating these methods to climatologists and experts in other applied fields such as engineering and finance

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- Thanks to the organizers and helpers of both conferences!